DIAS **Delhi Institute for Administrative Services**
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ALL INDIA TEST SERIES CSE-2024

Candidate 's Information

1. NAME:- Komal Punia
2. UPSC ROLL NO:- 0818290
3. MOBILE NO:- [redacted]
4. SUBJECT:- Nuclear Physics
5. DATE:- 18 July 2024

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Q.NO	MARKS
1.	28
2.	35
3.	33
4.	
5.	32
6.	30
7.	
8.	

Outstanding!

TOTAL MARKS	158/250

EXAMINER SIGNATURE

INVIGILATOR SIGNATURE

18 Pusa Road Karol Bagh New Delhi-110005 Ph:01140079000, 9350934622 e-mail diasindia123@gmail.com

प्रश्न संख्या
(Question No.)

U.P.S.C.

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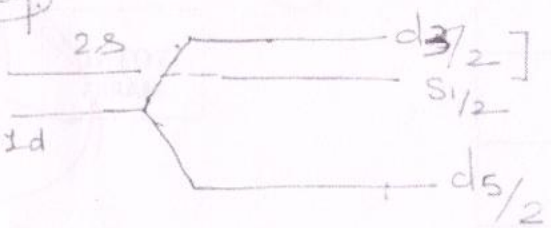
1 (a) Nuclear isomerism refers to different excited states of nuclei due to small energy difference (ΔE) and large ΔI (spin angular momentum) between shells.

Explaination by shell model

① Spin forbidden transitions by $\Delta I \neq 0$ by having energy levels close leads to metastable states

e.g.



] spin forbidden but ΔE small.

fig:- nuclei shells of 1d & 2S orbits

② Islands of isomers are observed near magic numbers as after them states are filled & have ΔE large.

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Few examples of nuclei showing nuclear isomers – ^{58}Co, ^{86}Rb etc.

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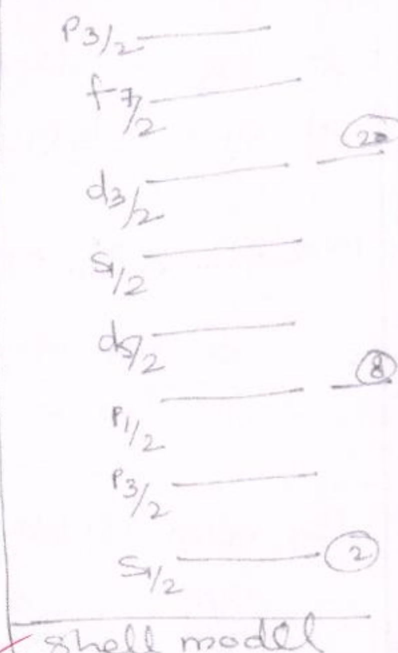
1(b) Predicting angular momenta (spin) & parities based on shell-model —



shell model

1) $\boxed{{}^{13}_{6}C}$

$Z = 6 \quad N = 7$

* Only odd nucleon contributes to I & parity.

So

$N = 7\ (1s_{1/2}^{2}\ 1p_{3/2}^{4}\ 1p_{1/2}^{1})$

$\Rightarrow \boxed{I^{\pi} = \left(\frac{1}{2}\right)^{-1}}$

2) ${}^{16}_{7}N \quad Z = 7,\ N = 9$

$Z\ [1s_{1/2}^{2}\ 1p_{3/2}^{4}\ 1p_{1/2}^{1}] \quad l_1 = 1,\ I_1 = 1/2$

$N\ [1s_{1/2}^{2}\ 1p_{3/2}^{4}\ 1p_{1/2}^{2}\ 1d_{5/2}^{1}] \quad l_2 = 2,\ I_2 = 5/2$

Using Nordheim's rules

$|l_1 + l_2 + I_1 + I_2| = \text{even } (6)$

$I = |I_1 - I_2| = |5/2 - 1/2| = 2$

So $\boxed{I^{\pi} = 2^{-1}}$

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3) ${}_{8}O^{17} \quad N = 9,\ Z = 8$

$N\ [1s_{1/2}^{2}\ 1p_{3/2}^{4}\ 1p_{1/2}^{2}\ 1d_{5/2}^{1}]$

$\Rightarrow \boxed{I^{\pi} = \left(\frac{5}{2}\right)^{+}}$

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1 (c) Semi-empirical mass formula is based on liquid-drop model of nucleus explains stability against β-decay.

$$M(Z,A) = ZM_H + (A-Z)M_n - \frac{1}{c^2}\left[a_v A - a_s A^{2/3} - \frac{a_c Z(Z-1)}{A^{1/3}} - \frac{a_a (N-Z)^2}{A} (\pm, 0) \frac{a_p}{A^{3/4}}\right]$$

for most stable isobar –

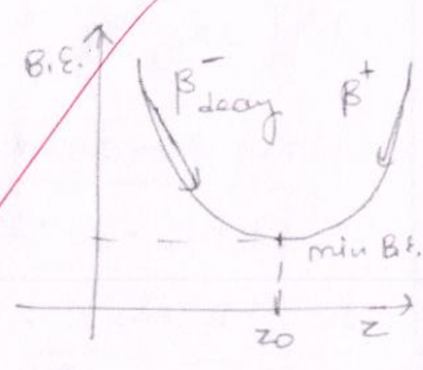
$$\left.\frac{\partial M}{\partial Z}\right)_{Z=Z_0} = 0 \ \& \ \left.\frac{\partial^2 M}{\partial Z^2}\right)_{Z=Z_0} > 0$$

$$\Rightarrow \boxed{Z_0 = \frac{4a_a + \frac{a_c}{A^{1/3}}}{2\left(\frac{4a_a}{A} + \frac{a_c}{A^{1/3}}\right)}}$$

Given $a_a = 19.3\,MeV$, $a_c = 0.583\,MeV$ \$ $A = 43$

$\Rightarrow Z_0 = 19.621$

$\Rightarrow \boxed{Z_0 \simeq 20}$ is most stable for $A = 43$



In case of odd-odd or even-even nuclei, we may also get multiple stable isobars.

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3(d)

1(e) Magnetic moment of nucleons is given by –

$\mu = \mu_{core} + \mu_{outer}$ [since $\mu_{core} = 0$ as fully filled shells] &

$\mu = \mu_{outer}$

$\boxed{\text{Calculation of magnetic moment}}$

$$\vec{\mu} = \left[g_p \vec{S}_p \frac{\mu_N}{\hbar} + g_n \vec{S}_n \frac{\mu_N}{\hbar} \right]$$

$$\mu_I = \frac{\vec{\mu} \cdot \vec{I}}{|I|^2} \cdot \vec{I} = \frac{\mu_N}{\hbar} \left[g_p \vec{S}_p \cdot \vec{I} + g_n \vec{S}_n \cdot \vec{I} \right] \frac{\vec{I}}{|\vec{I}|^2}$$

$$\mu_{I_Z} = m_I \frac{\mu_N}{\hbar} \left[g_p \left[\frac{|\vec{I}|^2 + |\vec{S}|^2 - |\vec{l}|^2}{2|\vec{I}|^2} \right] + g_n \left[\frac{|\vec{I}|^2 + |\vec{S}|^2 - |\vec{l}|^2}{2|\vec{I}|^2} \right] \right]$$

So here, $S = 1/2$ & $I = l + 1/2$ or $l - 1/2$

putting in above equation, | since $\boxed{m_I = I}$ for measurement

$\mu_{I_Z} = I + 2.28$ for $I = l + 1/2$

for proton $= I - \frac{2.28 I}{(I+1)}$ $I = l - 1/2$

$$\boxed{\begin{aligned} \mu_{I_Z} &= -1.91\,\mu_N \text{ for } I = l + 1/2 \\ \text{for neutron} &\quad \frac{1.91 I}{I+1} \text{ for } I = l - 1/2 \end{aligned}}$$

for ${}^{17}_{8}O$, $Z = 8$ & $N = \underline{\underline{9}}$

$N = 9 \left[1s_{1/2}^2 \; 1p_{3/2}^4 \; 1p_{1/2}^2 \; 1d_{5/2}^1 \right]$ This is $2 + 1/2$ $(l + 1/2)$ case.

$\Rightarrow$ $\boxed{\mu_{I_Z} \text{ for } {}^{17}_{8}O = -1.91\,\mu_N}$

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Q.5 a) γ-decay and Internal Conversion (I.C.) are both mechanisms of nuclear de-excitations.

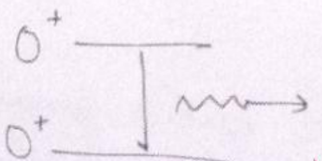
γ-decay selection rules & Internal Conversion

① $|\vec{J}_i - \vec{J}_f| \leq \Delta I \leq |\vec{J}_i + \vec{J}_f|$ $\quad \vec{J}_i$ = initial angular momentum

② No. of poles $= 2^{\Delta I}$ $\quad \vec{J}_f$ = final A.M.

If $\Delta I = 0 \Rightarrow$ Monopoles $\Rightarrow$ No γ-decay

eg.



× forbidden as $\Delta I = 0$

So this will proceed via Internal Conversion.

③ for γ-decay –

$\pi_f = (-1)^{l}(\pi_i)$ for Electric transitions

& $\pi_f = (-1)^{(l+1)}\pi_i$ for magnetic transitions.

Internal Conversion is knocking out of e^- from K-shell or L-shell by deexciting energy of nuclei.

$$\boxed{\text{Probability of I.C.} \propto z^3\left(\frac{e^2}{4\pi\epsilon_0 \hbar c}\right)^4 \left(\frac{2m_e c^2}{\Delta E}\right)^{L+3/2}}$$

So for large ΔI & small ΔE, I.C. is favoured over γ-decay.

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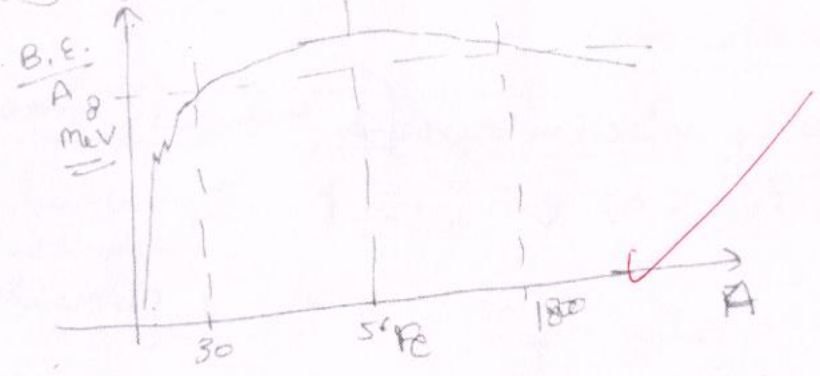
5(b) $B.E./A$ vs mass no. (A) curve helps in shydying nuclear properties.



So for $\boxed{30 \le A \le 170}$ $\frac{B.E.}{A} \simeq 8\,MeV$

Reasons -

① for $A \le 30$, size of nuclei small so surface energy reduces B.E.

② for $A \ge 180$, Coulomb repulsion reduces B.E.

③ Nuclear forces follows saturation property i.e. only applies to nearest neighbours.

④ Nuclear force is a short-ranged force.

Due to combined effect of above factors, $\frac{B.E.}{A}$ for $30 \le A \le 170$ is almost 8 MeV with ^{56}Fe being most stable nuclei

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Draw the individual energy contributions & then explanation is simple -

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5(c) Semi-empirical mass formula is based on liquid-drop model.

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} (\pm, 0) \frac{a_p}{A^{3/4}}$$

① Volume term ① Since $\frac{B.E.}{A} \simeq$ const.

$\Rightarrow B \propto A \Rightarrow \boxed{B = a_v A}$

It tends to bind nucleus together.

② Surface energy

surface nucleons faces attractive nuclear force from few directions only

$\Rightarrow B = -a_s A^{2/3}$ $\left[\text{as } B \propto \frac{4}{3}\pi R^2 \right.$ & $\left. R = R_0 A^{1/3} \right]$

③ Coulomb energy

repulsive Coulomb force reduces B.E.

$\Rightarrow B = -\frac{{}^Z C_2 e^2}{4\pi\epsilon_0 R} = -\frac{a_c Z(Z-1)}{A^{1/3}}$

where $a_c = \frac{3}{5} \frac{e^2}{4\pi\epsilon_0 R_0}$

④ Assymetry energy correctn

Symmetric configurations are more stable so assymetry uplifts energy.

$B = -a_a \frac{(N-Z)^2}{A}$ as $B \propto \frac{\Delta^2}{A}$

where $\Delta = \left(\frac{N-Z}{2}\right)$

⑤ Also pairing energy depends on no. of nucleons $(\pm, 0) \frac{a_p}{A^{3/4}}$.

+ for even-even
- for odd-odd & 0 for odd-even.

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5(d) Interactions violating following conservation laws -

1) Isotopic spin (I) - Weak Interactions & EM interactions violate conservat^n of I.
↳ Strong preserves.

2) Hypercharge - Weak violates
↳ EM & Strong preserves.

3) Lepton number - forms basic & laws, needs to be conserved by all 3 interactions.
EM Weak Strong → all preserves.

4) Charge Conjugation -
↳ weak violates
↳ Strong & EM conserves.

Based on above conservation laws, possibility of a reaction to occur can be determined.

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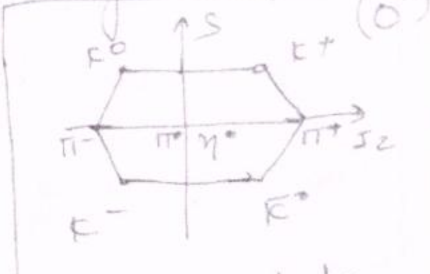
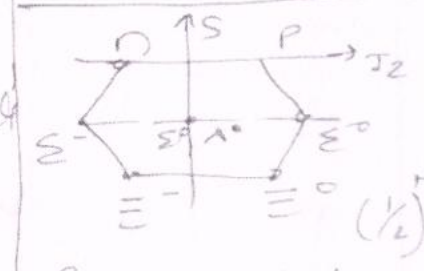
Q5 (c) Quark content of following composite particles —



Meson octate



Baryon octate.

1) π^+ (S = 0 ⇒ No strange particle)

$Q = +1 \Rightarrow u\bar{d}$)

So $\boxed{\pi^+ (|u\bar{d}\rangle)}$

2) K^+ (S = +1 ⇒ Antiparticle of strange particle

$q = +1$

$\Rightarrow |u\bar{s}\rangle$

$\Rightarrow \boxed{K^+ |u\bar{s}\rangle}$

3) Δ^{++} (S = 0 & Q = +2)

$\Rightarrow \boxed{\Delta^{++} | uuu\rangle}$

4) Σ^0 (S = −1 & Q = 0)

$\Rightarrow |s\bar{d}\rangle$ or $|\bar{d}s\rangle$

$\boxed{\Sigma^0 | \bar{d}s\rangle}$

5) Ω^- (S = −3, Q = −1)

$\Rightarrow \boxed{\Omega^- |sss\rangle}$

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Bound state of quarks are called

Hadrons — Mesons
— Baryons — Nucleons (S = 0)
— Hyperons (S ≠ 0)

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Q.3 a) Quarks are elementary particles which exists as bound states called Hadrons.

Quantum no. of quarks & antiquarks

Quantum No.	u	d	s	$\bar{u}$	$\bar{d}$	$\bar{s}$
I_z	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	0
Y	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
Q	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
S	0	0	-1	0	0	+1

1> Proton $[Q = +1, S = 0, I_z = +\frac{1}{2}]$

$\Rightarrow$ Proton [Baryon $\Rightarrow$ 3 quark bound state]

$\boxed{p = |uud\rangle}$

2> Neutron $[Q = 0, S = 0\ \ I_z = -\frac{1}{2}]$

$\hookrightarrow$ also 3 quark bound state

$\Rightarrow \boxed{n = |udd\rangle}$

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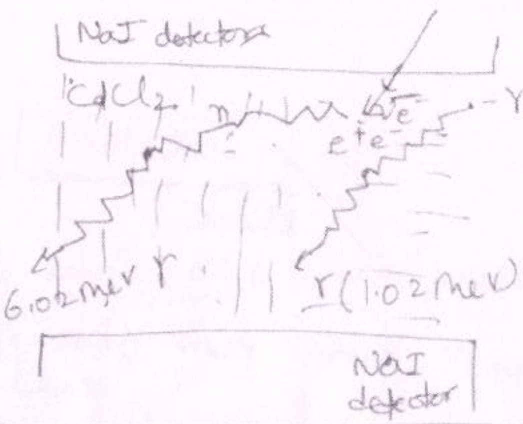
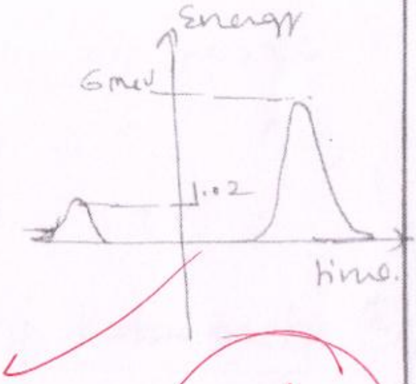
Q3 b) Neutrino detection solved mystery of angular momentum violation in β-decay.

Experimental evidence of Neutrino

Cowan & Reins using inverse β-decay reaction, by bombarding anti-neutrino on proton (from water) followed by detection of neutron in $CdCl_2$ confirmed existence of neutrinos.





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$$^1p + \bar{\nu}_e \longrightarrow {}^1n + e^+$$ Inverse β-decay reaction.

Helicity $H = \frac{\vec{p}\cdot\vec{S}}{|\vec{p}||\vec{S}|}$ — −1 (Neutrino), +1 (Antineutrino)

is a property which describes handedness and neutrino & helps prove parity violatⁿ in β-decay.

→ Goldhaber & Sunyar experiment measured helicity of neutrino as -1 ± 0.2

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Strange particles are elementary particles with certain characteristic properties.

Distinguishing properties of strange particles from non-strange ones –

① Production via strong interaction

eg. $p + \pi^0 \rightarrow K^+ + \Lambda^0$

$S = 0 \quad 0 \qquad +1 \quad -1$ [ΔS conserved]

② Decay via weak interaction

eg. $K^0 \rightarrow \pi^+ + \pi^-$

$S \quad +1 \qquad 0 \quad 0$ $\boxed{\Delta S \text{ violation}}$ here.

③ Associated production – two strange particles producti from non-strange ones

eg. $p + \pi^- \rightarrow K^0 + \Lambda^0$

$S = 0 \quad 0 \qquad +1 \quad -1$ (Associated productn)

④ Production is very fast (10^{-23} s) but decay slow (10^{-8} to few mins).

Examples of strange particles

Mesons
$K^+, K^0, K^-, \bar{K^0}$

Baryons
↓
Hyperons [$\Sigma^0, \Sigma^{-1}, \Sigma^{+1}$, Λ^0, Ξ^-, Ξ^0]

Gellmann - Nishijima relation establish relation between strangeness quantum no. with other quantum number like I_z, Q, B etc.

$$\boxed{Q = I_z + \frac{Y}{2}} \text{ or } \boxed{Q = I_z + \frac{B+S}{2}}$$

B = Baryon no.
I_z = Isospin component
Q = Charge
Y = Hypercharge.
S = Strangeness

Use in classification of elementary particle

- Non-strange, $S = 0$
  - eg. $n, p, \pi^-, \pi^0, \pi^+$, $\eta^0, \Delta^-, \Delta^0, \Delta^+, \Delta^{++}$
- Strange, $S \neq 0$
  - Mesons $\begin{bmatrix} K^+, K^0 \\ K^- \ \bar{K}^0 \end{bmatrix}$
  - Baryon → Hyperons $\begin{bmatrix} \Sigma^- \ \Sigma^0 \ \Sigma^+ \\ \Lambda^0 \ \Xi^- \ \Xi^0 \\ \Omega^- \end{bmatrix}$

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So Gellmann - Nishijima relation unfolded another criteria for elementary particle classification based on strangeness.

Also write about Symmetry & Conservation laws SU(2), SU(3)

Q3(d) Possibility of reactions can be determined by conservation of laws.

i) $n \longrightarrow p + e^- + \nu_e$

	n	p	e^-	ν_e	
q	0	+1	−1	0	✓
B	1	1	0	0	✓
S	0	0	0	0	✓
I	$1/2$	$1/2$	$1/2$	$1/2$	✓
I_z	$-1/2$	$1/2$	$-1/2$	$1/2$	X (violated)
L_e	0	0	1	1	X (violated)

So $\boxed{\text{forbidden reaction.}}$

ii) $\pi^- + p \longrightarrow \pi^- + \Sigma^+$

	π^-	p	π^-	Σ^+	
q	−1	+1	−1	+1	✓
B	0	1	0	1	✓
S	0	0	0	−1	X (violated)
I	1	$1/2$	1	1	X violated
I_z	−1	$1/2$	−1	+1	X violated

forbidden by strong & EM interact^n also ~~but~~ can't proceed via weak interact^n as decay of strange particle (as No strange particle in reactant)

So $\boxed{\text{forbidded reaction.}}$

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iii) $\Sigma^0 \longrightarrow \lambda^0 + \gamma$

Q	0	0	0	✓
B	1	1	0	✓
S	-1	-1	0	✓
I	1	0	0	× (violated)
I_z	0	0	0	✓

So $\boxed{\text{Allowed}}$ & occurs by EM interaction.

iv) $K^- + p \rightarrow n + \Lambda^0$

Q	-1	+1	0	0	✓
B	0	1	1	1	× (violation) of Baryon no. ⇓ forbidden
S	-1	0	0	-1	×
I	1/2	1/2	1/2	0	×
I_z	-1	1/2	-1/2	0	×

$\boxed{\text{Forbidden}}$

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for rxn to be allowed it should follow the basic conservatⁿ laws - mass, momentum, Spin, L_e, L_μ, L_τ, Baryon.

Q2 (a) Deuteron is most simplest nuclear structure of 1 proton & 1 neutron.

$\boxed{\text{Information from Deuteron problem}}$

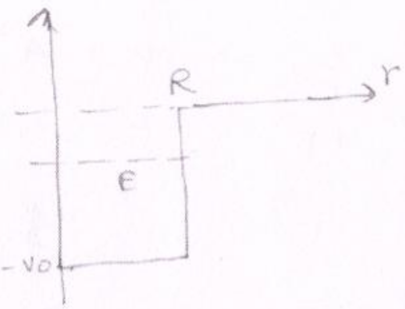
Experimental data –

① B.E. = 2.225 MeV ② Q = 0.00288 barn

③ Only 1 G.S. ④ $\mu = 0.574\,\mu_N$

⑤ Spin = 1, ⑥ Parity – Positive

Using schrondinger equation –



$$-\frac{\hbar^2}{2m}\nabla^2\psi + V(r)\psi = E\psi$$

Using $\psi(r,\theta,\phi) = \frac{U(r)}{r} Y_l^m(\theta,\phi)$

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[V(r) + \frac{l(l+1)\hbar^2}{2\mu r^2}\right]u = Eu$$

for G.S. $\Rightarrow l = 0$ & $V(r) = -V_0$

$$\Rightarrow \frac{d^2u}{dr^2} = -k_1^2 u \quad \text{where } k_1 = \sqrt{\frac{2m(V_0+E)}{\hbar^2}}$$

for $r \leq R$

$$\frac{d^2u}{dr^2} = k_2^2 u \Rightarrow k_2 = \sqrt{\frac{-2mE}{\hbar^2}}$$

for $r \geq R$

So solutions are –

$u = A\sin k_1 r \quad r \leq R \quad$ [B=0 using B.C.]

$u = Ce^{-k_2 r} \quad r > R \quad$ [as $r \to \infty$ $\psi \to 0$]

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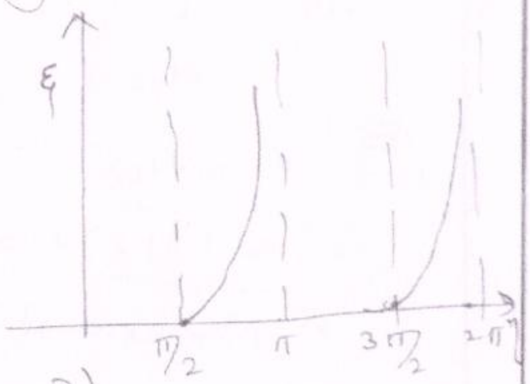
from solutions, we get a transcental equation as

$$-k_1 \cot k_1 r = k_2$$

Let $\eta = k_1 r$ & $\xi = k_2 r$

$$\boxed{\eta \cot \eta = -\xi}$$



also

$$\eta^2 + \xi^2 = \alpha^2 = \left(\frac{2mV_0 r^2}{\hbar^2}\right)$$

for g.s. (atleast 1 solution)

$$\alpha \geq \frac{\pi}{2}$$

$$\Rightarrow \frac{2mV_0 R^2}{\hbar^2} \geq \frac{\pi^2}{4}$$

$$\Rightarrow \boxed{R^2 \geq \frac{\pi^2 \hbar^2}{8\mu V_0}}$$

where μ is reduced mass $\mu = \frac{m_p}{2}$

So $\boxed{R = \sqrt{\frac{\pi^2 \hbar^2}{8\mu V_0}}}$ is radius of deuteron

for $V_0 = 30$ MeV, $\mu = \frac{m_p}{2}$

$\boxed{R \simeq 2.1 \text{ fm}}$ size of Deuteron.

Information about nuclear forces from deuteron -

① Non-central nature of nuclear force

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as $\mu = 0.8574\ \mu_N$
while $\mu_{1s} = 0.879\ \mu_N$
$\mu_{1d} = 0.312\ \mu_N$

$\mu = 0.96\ \mu_{1d} + 0.04\ \mu_{1s}$
i.e. 96% (1s) state & 4% 1d state
⇒ Non-central force.

(2) Nuclear forces are tensor in nature
(3) Short ranged nature of nuclear force
(4) ~~...~~ Charge independence and charge symmetric nature.
(5) Attractive force nature.

So Deuteron study helped understand nuclear force nature.

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Q.2 b) Nuclear shell model models the nucleons in nuclear shells exhibiting magic numbers like atomic shells.

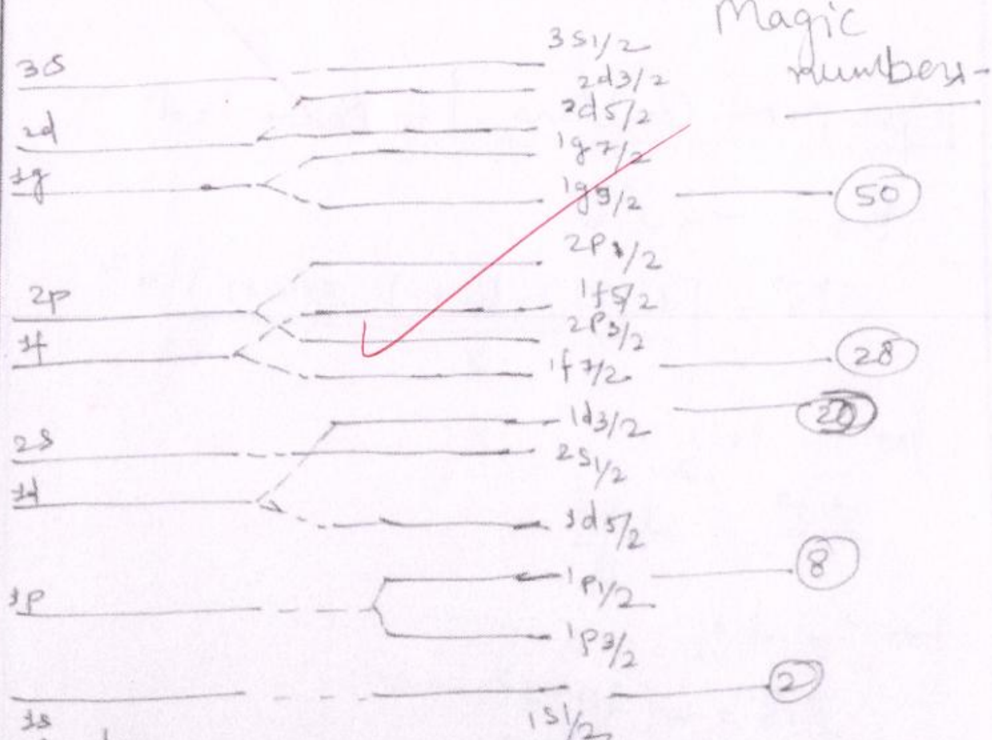
Shell model assumptions

① Strong inverted spin-orbit interaction $(-\alpha \vec{L}\cdot\vec{S})$

② Weak potential due to other nucleons on a nucleon.

③ Existence of nuclear shells.

Shell model Nuclear shells as per shell model are –



Woods-Saxon potential

$-\alpha \vec{L}\cdot\vec{S}$
Inverted spin-orbit coupling.

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So magic nos represents fully filled shells so are more stable.

$\boxed{\text{Evidences}}$ —

(1) $^{208}_{82}Pb$ (doubly magic) is extra stable
↳ end product of radioactive series.

(2) $^{212}_{84}Po$ is strong α emitter.

$$^{212}_{84}Po \longrightarrow {}^{208}_{82}Pb + {}^{4}_{2}He$$

(3) Islands of isomers near shell closure

(4) Cross sectⁿ of neutron capture near fully filled shell is low.

(5) Kinks in B.E. curve at magic numbers ⇒ extra stability.

$\boxed{\text{Spin-orbit Coupling}}$ — Inverted

$$H_{ls} = -\alpha \vec{l}\cdot\vec{s}$$

$$\vec{l}\cdot\vec{s} = \left[\frac{J(J+1) - l(l+1) - s(s+1)}{2}\right]\hbar^2$$

for $j = l + 1/2$

$$\vec{l}\cdot\vec{s} = \frac{l\hbar^2}{2}$$

for $j = l - 1/2$

$$\vec{l}\cdot\vec{s} = -\frac{(l+1)\hbar^2}{2}$$

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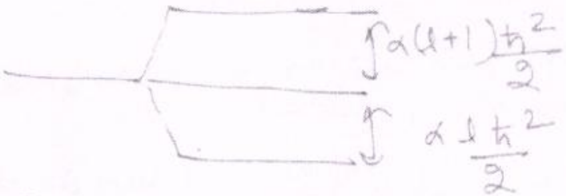

$$\Delta E = \frac{\alpha(2l+1)\hbar^2}{2} = \underline{\alpha'(2l+1)}$$

Given $\alpha'(2l+1) = 8$ MeV

Calculating strength by assuming strength determinatⁿ b/w ${}^1f_{7/2}$ & ${}^1f_{5/2}$ $l=3$

$= (2\times3+1)\alpha' = 8$

$\Rightarrow \boxed{\alpha' = 8/7 \text{ MeV}}$

for ${}^1p_{3/2}$ & ${}^1p_{1/2}$ $l=1$

$\Rightarrow (2\times1+1)\alpha' = 8$

$\Rightarrow \alpha' = \underline{8/3} \text{ MeV}$

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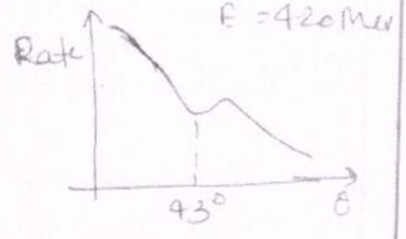
Electronic methods] use of e^- to determine nuclear radius.

① Electron deflection method

↳ e^- deflected by nucleus –



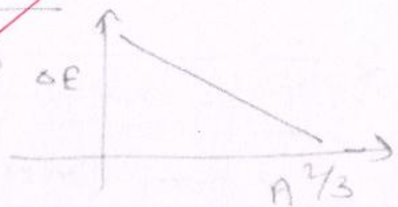
$\Rightarrow 2d\sin\theta = 1.22\lambda$

$\Rightarrow d = \dfrac{1.22\lambda}{2 \times \sin 43°}$
(diameter)

from here nuclear radius ≃ 2 fm.

② Isotopic substitution

$\boxed{\Delta E \propto A^{2/3}}$ So



from here, for different isotopes, nuclear radius can be determined.

③ muonic x-ray method – use of muons instead of e^- in scattering to determine nuclear radius.

Mirror nuclei method of radius determinat^n

$^{A}_{Z}X \rightarrow \, ^{A}_{Z-1}Y + e^+ + \nu_e$ [β^+ decay rxn, $^{A}_{Z}X$ & $^{A}_{Z-1}Y$ are mirror nuclei]

Using semi-empirical mass formula –

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$$Q = \begin{bmatrix} \text{Mass of} & - & \text{Mass of} \\ \text{reactants} & & \text{Product} \end{bmatrix} c^2$$

$$= \left\{ Z M_H + (A-Z)M_n - \left[a_v A - a_s A^{2/3} - \frac{a_c Z(Z-1)}{A^{1/3}} - \frac{a_a (N-Z)^2}{A} \right] \right\} -$$

$$\left\{ (Z-1)M_H + (A-Z+1)M_n - \left[a_v A - a_s A^{2/3} - \frac{a_c (Z-1)(Z-2)}{A^{1/3}} - \frac{a_a (N-Z)^2}{A} \right] \right\} - m_e c^2$$

$$Q_{\beta^+} = M_n - M_H + \frac{a_c (2Z-1)}{A^{1/3}}$$

$$\Rightarrow \boxed{Q_{\beta^+} = a_c A^{2/3} - [M_H - M_n]} \quad \left\{ \text{as } Z + Z - 1 = A \right\}$$

from Q_{β^+}, we can get a_c $\left(a_c = \frac{3}{5} \frac{e^2}{4\pi\epsilon_0 R_0} \right)$

So from a_c, R_0 can be determined.

Hence mirror nuclei can be used to determine nuclear radius.

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Q6(b) What interactions exhibits parity violation i.e. mirror image of a reaction doesn't represent physical reality.

Not possible to detect parity violat^n by observing only β-decay rate —

① β-decay rate is symmetric for both β^+ & β^- i.e. for both cases, it is same.

② β-decay rate doesn't depend on polarisation → which is often used to determine parity violation.

Draw the diagrams for γ-Ray asymmetry & β-asymmetry

③ decay rate only signifies no. of counts observed in detectors. It doesn't deal with polarity or helicity ⇒ so no use in parity violat^n checking.

$\boxed{\text{Parity violat}^n}$ can be detected by polarised ^{60}Co β-emission as done by Cowan & Reins.

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Q 4(c) $^{13}_{6}C \longrightarrow ^{13}_{7}N + \beta + \bar{\nu}_\beta$ (β-decay)

Given max. energy of β-particle = 1.202 MeV

Q-value of reaction refers to kinetic energy supplied to products.

So max. energy of β-particle ⇒ neutrino at rest = Q value of rxn.

$$Q = \left[m(^{13}_{6}C) - m(^{13}_{7}N) - m_\beta - m_{\nu_\beta}\right]c^2$$

as $Q = \left[\begin{pmatrix}\text{mass of}\\ \text{reactant}\end{pmatrix} - \begin{pmatrix}\text{mass of}\\ \text{product}\end{pmatrix}\right]c^2$

$$Q = \left[13.003354 \times 931.5 - 0.511\right] - m(^{13}_{7}N)c^2$$

$$m(^{13}_{7}N)c^2 = 12112.62425 - 0.511 - 1.202$$

$$= 12110.91125 \text{ MeV}$$

$$\Rightarrow \boxed{m(^{13}_{7}N) = 13.00151\ u}$$

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Here assumption is that max. energy of β-particle is its Kinetic Energy (& not total energy i.e. rest mass energy + K.E.)

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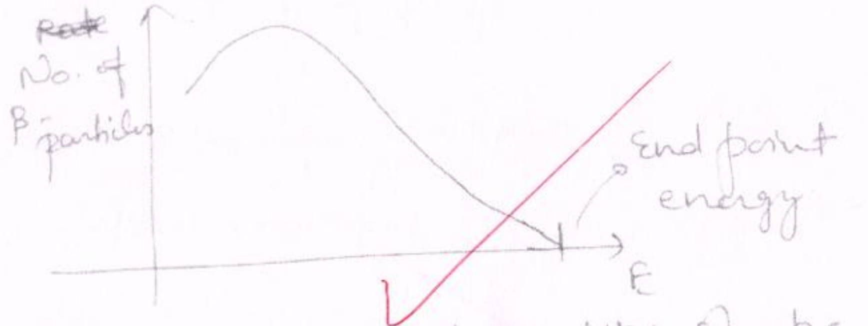
β- decay spectra is usually of shape -



This spectra is unlike like spectra of α-particle emission bcz of presence of neutrino emissions alongwith β- particles.